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A Comparative Study of *Origami* Inspired Folded Plates

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Abstract

Origami is an old art of paper folding. From mechanical point of view origami can be defined as a folded structure. In the present paper a comparative study of four origami inspired folded plate structures is presented. Longitudinal, facet, egg-box and Miura-ori origami modules are used for the analysis. The models are based on six-parameter shell theory with the use of the finite element method. Convergence analysis of each module is presented. Numerical study of roof folded plates in oriented to the comparison of maximal displacements and stresses in the structures. Some parametric analysis is also presented.

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Keywords: folded plate; origami; finite element modelling

1. Introduction

Folded plates is an attractive solution for architects. There are a lot of interesting engineering structures of this kind, some of them are based on the concept of *origami* - old art of paper folding dated for the 7th century. From mechanical point of view the correct theory to describe folded plates is six parameter shell theory with three displacements and three rotations in the displacement field. The third rotation is necessary because of folding the structure. Each fold is flat, so the equations of the theory can be simplified. For numerical analysis it is necessary to use the finite element method because of complex character of the structures. Finite elements with six d.o.f. per node are to be used.

The present paper is dedicated to the comparatives study of several patterns of folded plates, to answer the question if those structures are also attractive from mechanical point of view. The most challenging task is to develop the effective technique for efficient computation of structures with a lot of folds. The authors tried to select the most attractive (from engineering point of view) *origami* based folded plate pattern. The analysis is done with the use of the Abaqus professional software.

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2. *Origami* inspired folded plates

Origami is an old art of paper folding developed in Japan with origins in China. The term is a combination of two Japanese words: 'oru' – to fold and 'kami' – paper. There are several typical folds in the *origami* art. Some of them are presented in Fig. 1a: 'mountain fold', 'valley fold', 'swivel fold'. After change of the direction of mountain fold one can receive the 'inside reverse fold' or 'outside reverse fold' (Fig.1b). The other possibilities are 'squash fold' and 'sink fold' (Fig.1c). A detailed description of *origami* folds and patterns can be found in [3]. *Origami* is an inspiration for engineers in the fields of civil engineering, architecture, *biotechnology*, medicine, space engineering and other technical applications [5-8].

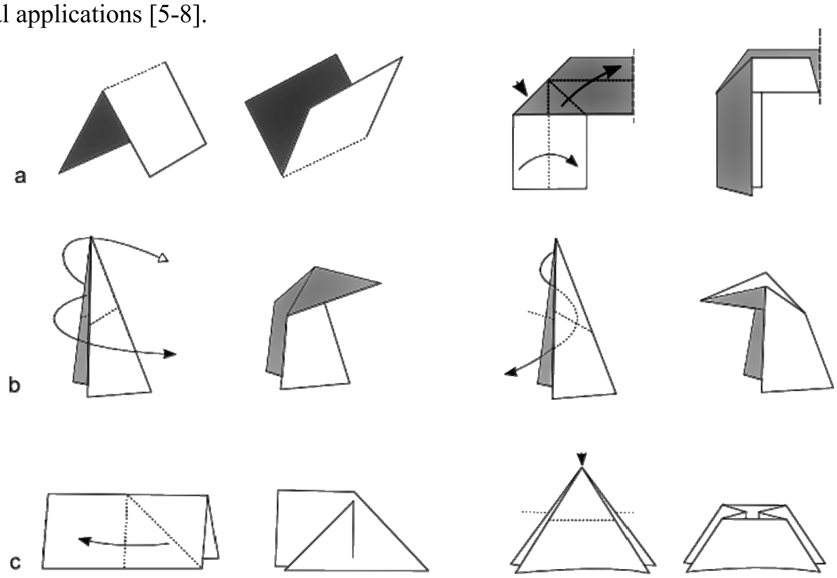

Fig. 1. *Origami* folds: (a) 'mountain fold', 'valley fold', 'swivel fold';
(b) 'inside reverse fold', 'outside reverse fold'; (c) 'squash fold', 'sink fold'.

From mechanical point of view origami can be defined as a folded structure [1, 3]. Folded plate is defined as a 3D folded structure placed within two planes. Mathematical model of each fold is six-parameter flat shell theory [2] with the curvature tensor $b_{\alpha\beta} = 0$. Let us consider a flat shell of thickness *h*. Displacement field is described by three linear displacements u_α, w of middle surface and three rotations ϕ_α, ψ. The following equations are to be valid (see [2] for details) with the Einstein sum convention in use:

- geometrical relations ($\gamma_{\alpha\beta}$, $\kappa_{\alpha\beta}$, $\gamma_{\alpha 3}$, $\kappa_{\alpha 3}$, γ_{33} - strain components, $\in_{\alpha\beta}$ - Ricci symbol):

$$\begin{aligned} &\gamma_{\alpha\beta} = u_{\alpha,\beta} - \in_{\alpha\beta} \psi, \ \ \kappa_{\alpha\beta} = \phi_{\alpha,\beta}, \\ &\gamma_{\alpha 3} = \phi_\alpha + w_{,\alpha}, \ \ \kappa_{\alpha 3} = \psi_{,\alpha}, \ \ \gamma_{33} = \psi, \end{aligned} \tag{1}$$

- constitutive equations for isotropy ($N_{\alpha\beta}$, $M_{\alpha\beta}$, $N_{\alpha 3}$, $M_{\alpha 3}$ - internal forces, E – Young modulus, ν - Poisson ratio):

$$\begin{aligned} &N_{\alpha\beta} = B^0_{\alpha\beta\lambda\mu} \ \gamma_{\lambda\mu}, \ \ M_{\alpha\beta} = \frac{h^2}{12} B^0_{\alpha\beta\lambda\mu} \ \kappa_{\lambda\mu}, \\ &N_{\alpha 3} = k^2 B^0_{\alpha 3\beta 3} \ \gamma_{\beta 3}, \ \ M_{\alpha 3} = \frac{h^2}{12} l^2 B^0_{\alpha 3\beta 3} \ \kappa_{\beta 3}, \end{aligned} \tag{2}$$

$$B^0_{\alpha\beta\lambda\mu} = Gh\left(a_{\alpha\lambda} a_{\beta\mu} + a_{\alpha\mu} a_{\beta\lambda} + \lambda a_{\alpha\beta} a_{\lambda\mu}\right), \ \ B^0_{\alpha 3\beta 3} = Gha_{\alpha\beta}, \tag{3}$$

$$G = \frac{E}{2(1+\nu)},\ \lambda = \frac{2\nu}{1-\nu},\ k^2 = \frac{5}{6},\ l^2 = \frac{7}{10}, \tag{4}$$

- equilibrium equations ($f_\beta,\ f_3,\ m_\beta,\ m_3$ - external loads)

$$\begin{aligned}
&N_{\alpha\beta,\alpha} + f_\beta = 0,\\
&N_{\alpha 3,\alpha} + f_3 = 0,\\
&M_{\alpha\beta,\alpha} - N_{\beta 3} + m_\beta = 0,\\
&M_{\alpha 3,\alpha} + \in_{\alpha\beta} N_{\alpha\beta} + m_3 = 0.
\end{aligned} \tag{5}$$

The equations of folded plates are solved by the finite element method.

3. Numerical analysis

Numerical analysis is done with the use of Abaqus programme. According to the above consideration shell finite elements are used for numerical calculations. Selection of the correct element with convergence analysis for *origami* inspired modules are presented in [4]. 4-noded S4R shell element with 6 d.o.f. per node with hourglass control is used for further calculations. The element can be used for thin as well as moderately thick folded plates.

The subject for numerical analysis in the present paper is a folded plate roof on a square $6m \times 6m$ made of steel (*E = 210 GPa v =0.2*) with constant material volume 0.81 m^3. 36 modules are used for each *origami* inspired folded plates presented in Fig. 2-5 with *H = 1 m*. The results are compared with deformation of the plate with constant thickness h = 0.0225 m.

Constant thickness plate, as well as folded plates with longitudinal, Facet, Eggbox and Mura-ori patterns are clamped on two opposite edges with free boundary conditions on two other edges. For facet and Miura-ori patterns clamped lines or clamped main on the edges points were applied.

Parametric study of the Eggbox and Miura-ori patterns was also studied but are not included into this paper. The other interesting aspect is to introduce semi-rigid or flexible connections on the borders of flat plates. Other interesting analysis is a truss model of the *origami* structure. For the Eggbox and Miura-ori pattern it is possible to arrange the self-stress state and the infinitesimal mode are observed. Analysis of those aspect is a current work of the authors.

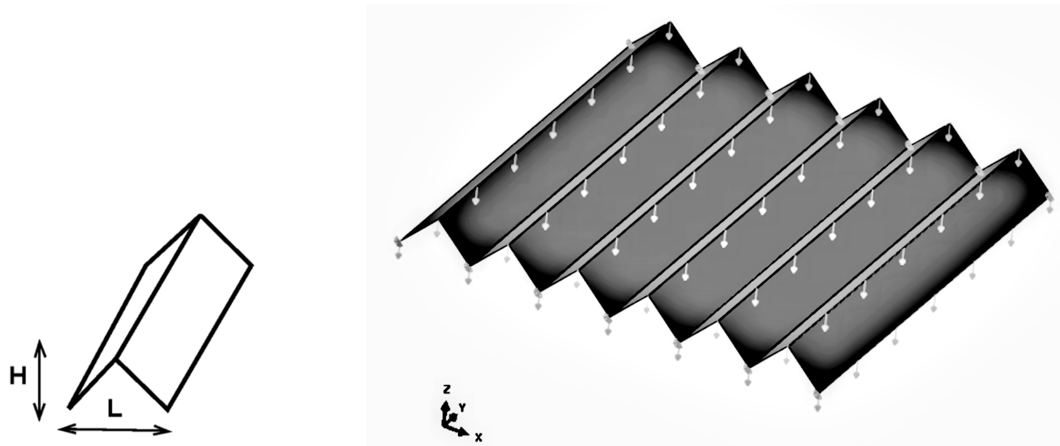

Fig. 2. Longitudial pattern.

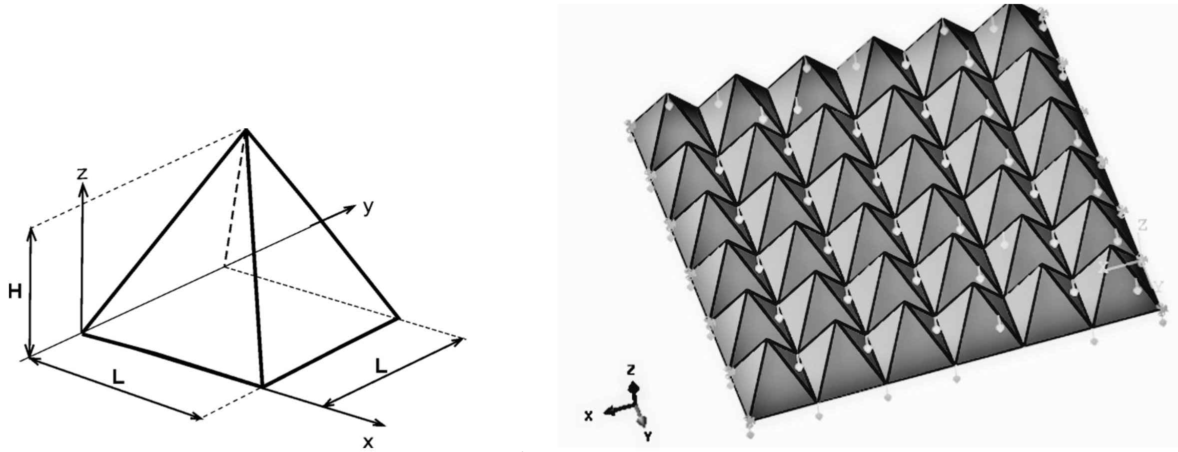

Fig. 3. Facet pattern.

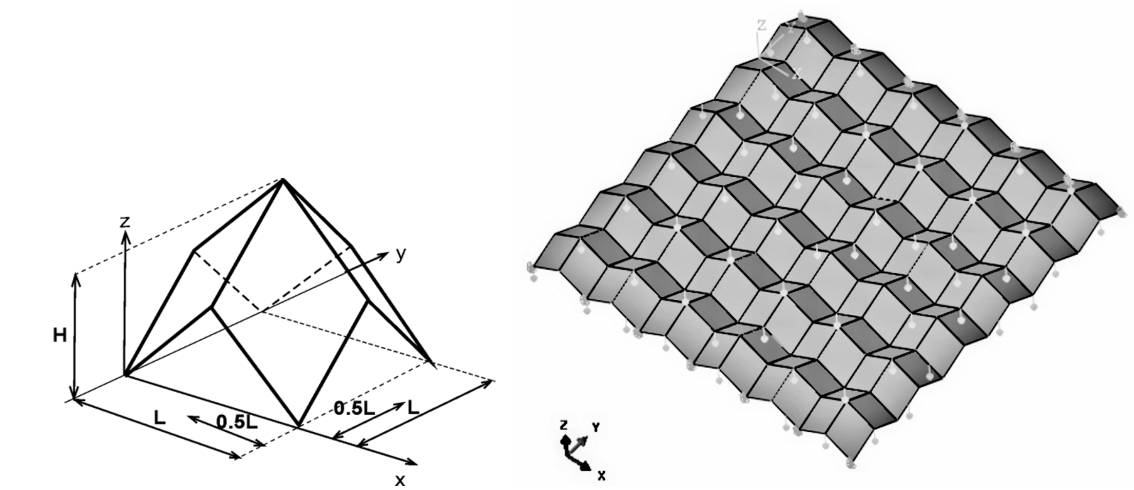

Fig. 4. Eggbox pattern.

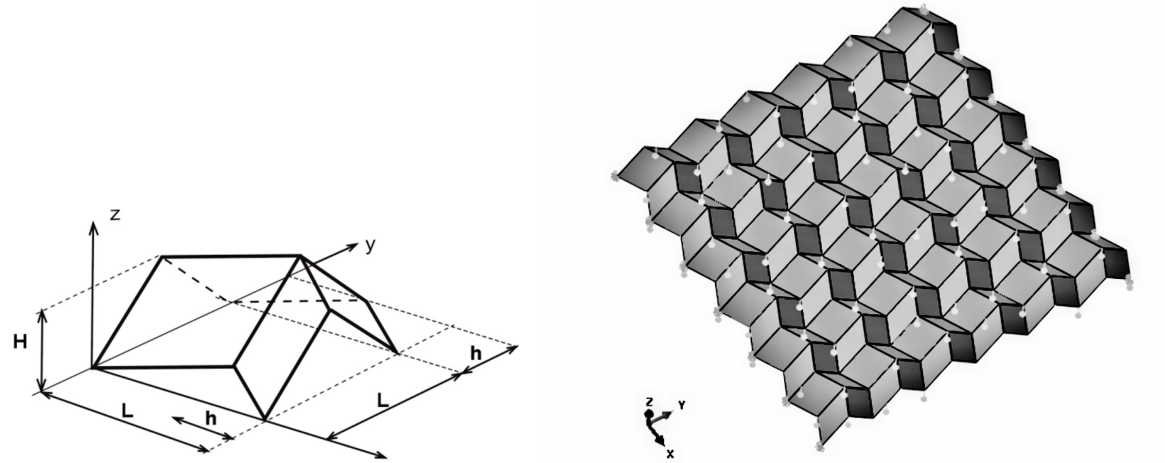

Fig. 5. Miura-ori pattern.

Characteristic deformations and Von Mises stress maps for the above structures are presented in Fig. 6-9.

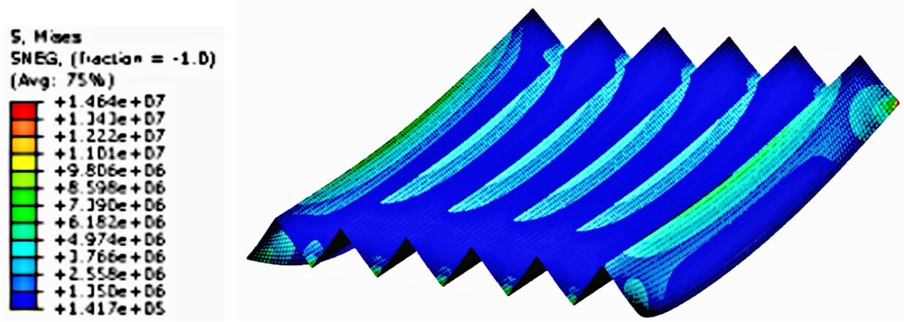

Fig. 6. Displacements and Von Mises stresses for longitudinal pattern folded plate.

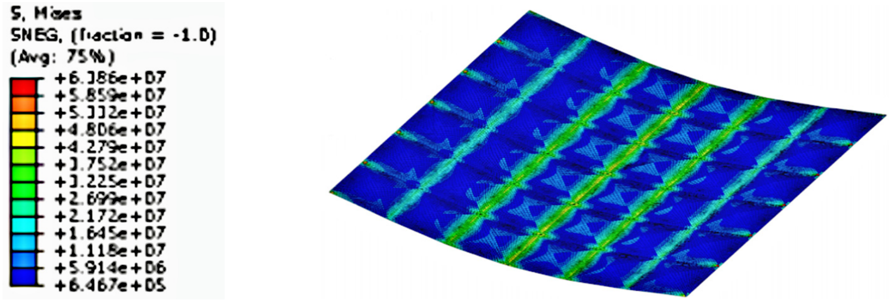

Fig. 7. Displacements and Von Mises stresses for Facet pattern folded plate.

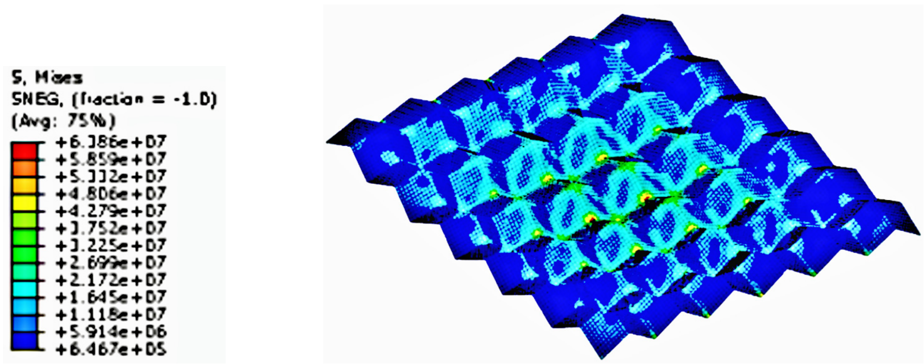

Fig. 8. Displacements and Von Mises stresses for Eggbox pattern folded plate.

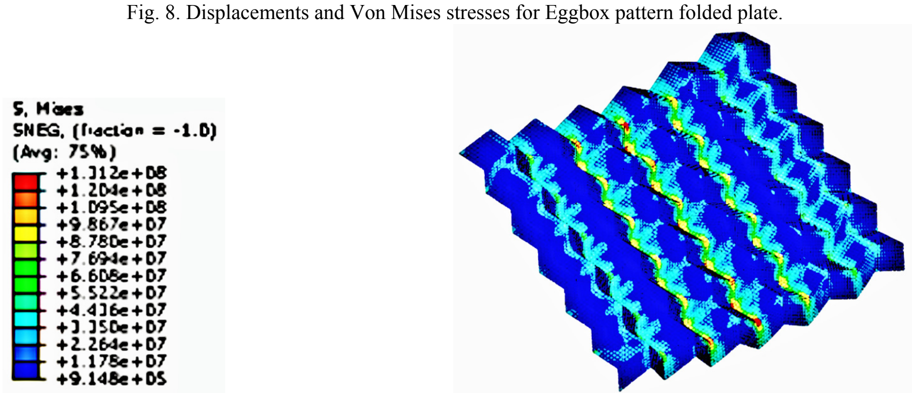

Fig. 9. Displacements and Von Mises stresses for Miura-ori pattern folded plate.

Selected results of all examples are compared in the Tab. 1.

Table 1. Comparison of results for 6mx6m flat and folded plates of constant material volume

Pattern	Boundary conditions	Thickness [m]	Total area of folds [m2]	Number of finite elements	Max vertical displacement [m]	Von Mieses stress in the middle of plate [MPa]
Flat plate	Clamped lines opposite sides	0.0225	36.000	7396	0.14305	85.892
Longitudial	Clamped points opposite sides	0.0101	80.499	7920	0.00005	1.530
Facet	Clamped lines opposite sides	0.0101	80.499	7102	0.11946	120.705
	Clamped points opposite sides	0.0101	80.499	7102	0.11958	120.684
Eggbox	Clamped lines opposite sides	0.0130	62.354	14400	0.01085	25.439
Miura-ori	Clamped lines opposite sides	0.0075	108.263	7200	0.00261	9.899
	Clamped points opposite sides	0.0075	108.263	7200	0.00376	13.569

4. Conclusions

The present paper is dedicated to the analysis of folded plates inspired by the *origami* art. Six-parameter moderately thick shell theory is used with the finite element formulation. Numerical analysis is done with the use of Abaqus software. Various patterns of folded plates are compared for the square roof constant material volume. Maximum displacements and Von Misses stresses are compared. The best results are obtained for the longitudinal pattern. Within typical *origami* patterns very promising results are obtained for the Miura-ori. To conclude, the folded plate concept seems to be very attractive from engineering point of view as well as interesting for architects.

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